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Geometrically nonlinear transient analysis of laminated composite shells using the finite element method

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Abstract

The nonlinear transient response of composite shells with/without cutouts and initial geometric imperfection is investigated using the finite element method. The present formulation considers doubly curved shells incorporating von Kármán type nonlinear strains into the first order shear deformation theory. The analysis is carried out using quadratic C^0 eight-noded isoparametric element. The governing nonlinear equations are solved by using the Newmark average acceleration method in the time integration in conjunction with modified Newton–Raphson iteration scheme. The validity of the model is demonstrated by comparing the present results with those available in the literature. Parametric studies are carried out varying the radius of curvature to width ratio and amplitude of initial geometric imperfection of laminated composite cylindrical, spherical and hyperbolic paraboloid shells with/without cutouts.

1. Introduction

In recent years, composite shell structures are increasingly used in aerospace, civil and other industries because of their superior mechanical properties such as high strength-to-weight ratio, high stiffness-to-weight ratio, excellent fatigue and corrosion resistance properties, and ease in fabrication. Very often these composite shells undergo large deformation when subjected to severe dynamic loading conditions necessitating the study of their vibration behavior in the nonlinear domain. Cutouts are often provided due to the practical requirements of lightening the structure, venting or providing accessibility to other parts of the structure. Effects of cutouts are likely to be considerable when the structure undergoes large deformation. During the fabrication process or later in the service of laminated shells, it is possible that unavoidable geometric imperfection may be present. Thus, it is necessary to investigate the nonlinear transient response of composite shells with/without cutouts and geometric imperfection.

The classical theory based on Love–Kirchhoff assumptions under predicts deflection and over predicts natural frequency and buckling load. Due to high ratios of in-plane elastic moduli to transverse shear moduli

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for most of the composite laminates, the transverse shear deformations for composite laminates are more pronounced compared to those of isotropic shells. Thus, more accurate theories became necessary including the effects of transverse shear deformation in order to have a reliable analysis and safe design. To counter the drawbacks of classical theories, shear deformable theories were established [1]. The first order shear deformation theory (FSDT), where a plane straight and normal to the middle surface remains straight after the deformation but not necessarily remains normal to it, was extensively employed in the analysis of moderately thick laminates. The FSDT together with proper shear correction factors can predict most of the structural behavior adequately [2]. Moreover, in comparison with higher order theories, the FSDT has the advantage of being simpler and involves less computational cost.

The dynamic behavior of shells is governed by nonlinear partial differential equations. Among the different nonlinear analysis methods, the finite element method [3,4] has been widely used in solving nonlinear problems because of its easy implementation with any boundary condition and arbitrary geometry. During the last decade, several finite element computational models were developed for the nonlinear analysis of shell structures. Brank et al. [5] presented a simple nonlinear four-noded shell finite element for thin multilayered elastic shells. To and Wang [6] investigated geometrically nonlinear behavior of laminated composite plate and shell structures by developing a hybrid strain based laminated composite triangular shell finite element. Balah and Al-Ghamedy [7] presented the finite element formulation of a four-noded isoparametric laminated shell element based on a third order shear deformation theory with finite rotations. Vu-Quoc and Tan [8] developed optimal solid shells for nonlinear analysis of multilayered composites. Arciniega and Reddy [9] presented a tensor based finite element formulation for geometrically nonlinear analysis of shell structures. Han et al. [10] formulated a nine-noded modified first order shear deformable element for geometrically nonlinear analysis of laminated composite thin plates and shells with large displacement and small strain.

Different authors reported investigations of nonlinear transient responses of laminated composite shells. Nath and Alwar [11] studied the nonlinear dynamic response of shallow spherical shells with and without damping subjected to various types of transient loads using Chebyshev series expansion. Chao and Reddy [12] developed a degenerated three-dimensional element based on the total Lagrangian description of the motion to analyze nonlinear bending, free vibration and transient response of laminated composite shells. Reddy and Chandrashekhara [13] presented geometrically nonlinear transient analysis of laminated composite shells using the finite element method. Saigal and Yang [14] studied the nonlinear dynamic responses of shells using a four-noded, 48 degrees of freedom, curved, quadrilateral thin shell element. Wu et al. [15] investigated the free and forced nonlinear responses of thin laminated composite shells using a higher order curved shell finite element. Bagchi et al. [16] developed an incremental finite element matrix formulation for the solution of nonlinear dynamic problems of moderately thick plates and shells. Ganapathi and Varadan [17] investigated the nonlinear transient response of anisotropic laminated plates/shells using a field-consistent shear flexible element. Kant and Kommineni [18] presented a C^0 continuous finite element formulation of a higher order shear deformation theory for predicting the linear and geometrically nonlinear transient responses of laminated composite and sandwich shells. Kumar and Singh [19] presented geometrically linear and nonlinear dynamic analyses of isotropic and laminated composite cylindrical panels using Bezier surface patches. To and Wang [20] developed a hybrid strain based flat triangular shell element for geometrically nonlinear transient analysis of laminated composite plates and shells. Naboulsi and Palazotto [21] presented three nonlinear static-dynamic finite element formulations, namely, simplified large rotation, large displacement large rotation and Jaumann analysis, for composite shells. Kundu and Sinha [22] investigated geometrically nonlinear transient response of laminated composite doubly curved shells using nine-noded isoparametric composite shell element including the first order shear deformation. Nanda and Bandyopadyay [23] studied transient behavior of laminated composite doubly curved shells with cutouts subjected to different types of transient loads.

The above review reveals that exhaustive investigations were carried out on the geometrically nonlinear transient response of laminated composite shells. However, the same for shells with cutouts and initial geometric imperfection are not addressed. In this paper, the finite element method is employed to study the geometrically nonlinear transient response of laminated composite shells with/without square cutouts and initial geometric imperfection.

2. Theory and formulation

A laminated composite shell panel with cutout (Fig. 1) is considered with thickness h. Each of the thin lamina can be oriented at an arbitrary angle θ with reference to the x-axis. The displacement field is assumed to be of the form

$$U(x,y,z) = u(x,y) + z\theta_x(x,y), \quad V(x,y,z) = v(x,y) + z\theta_y(x,y) \quad \text{and} \quad W(x,y,z) = w(x,y) \tag{1}$$

where U, V and W are the displacements at any point (x,y,z) of the shell in x, y and z directions, respectively; u, v and w are the associated mid-plane displacements; and θ_x and θ_y are the rotations about y and x axes, respectively.

Here, strain–displacement relations of the shear deformable theory of doubly curved shells that includes von Kármán type geometric nonlinearity are derived. Sanders' nonlinear strain–displacement relations [4,24] associated with the displacement field are used. The strain–displacement relations are given by

$$\varepsilon'_x = \varepsilon_x + z\kappa_x, \quad \varepsilon'_y = \varepsilon_y + z\kappa_y, \quad \gamma'_{xy} = \gamma_{xy} + z\kappa_{xy}, \quad \gamma'_{xz} = \gamma_{xz}, \quad \gamma'_{yz} = \gamma_{yz} \tag{2}$$

where the strains and curvatures of the middle surface are related to the displacements by

$$\varepsilon_x = \partial u/\partial x + w/R_x + 1/2(\partial w/\partial x - u/R_x)^2,$$

$$\varepsilon_y = \partial v/\partial y + w/R_y + 1/2(\partial w/\partial y - v/R_y)^2$$

$$\gamma_{xy} = \partial u/\partial y + \partial v/\partial x + 2w/R_{xy} + (\partial w/\partial x - u/R_x)(\partial w/\partial y - v/R_y)$$

$$\gamma_{xz} = \theta_x + \partial w/\partial x - u/R_x - v/R_{xy}, \quad \gamma_{yz} = \theta_y + \partial w/\partial y - v/R_y - u/R_{xy}$$

$$\kappa_x = \partial\theta_x/\partial x, \quad \kappa_y = \partial\theta_y/\partial y, \quad \kappa_{xy} = \partial\theta_x/\partial y + \partial\theta_y/\partial x + C_0(\partial v/\partial x - \partial u/\partial y) \tag{3}$$

Here R_x, R_y and R_{xy} are the usual radii of curvature. Term $C_0 = 0.5(1/R_y - 1/R_x)$ is the result of Sanders' theory, which accounts for zero strain for rigid body motion.

The stress–strain equations of the shell are given as follows:

$$\{F\} = [D]\{\varepsilon\} \tag{4}$$

where the stress and strain vectors are

$$\{F\} = \{N_x \; N_y \; N_{xy} \; M_x \; M_y \; M_{xy} \; Q_x \; Q_y\}^{\mathrm{T}},$$

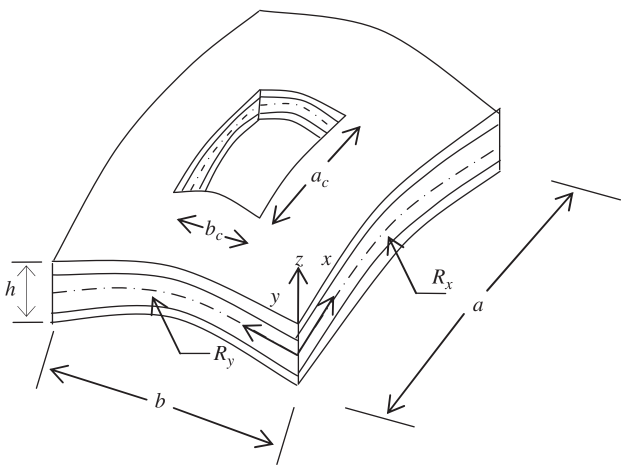

Fig. 1. Laminated composite doubly curved shell with cutout.

$$\{\varepsilon\} = \{\varepsilon_x \;\; \varepsilon_y \;\; \gamma_{xy} \;\; \kappa_x \;\; \kappa_y \;\; \kappa_{xy} \;\; \gamma_{xz} \;\; \gamma_{yz}\}^{\mathrm{T}}$$

The elements of stiffness matrix $[D]$ are furnished by Reddy [1].

2.1. Finite element formulation

An eight-noded isoparametric element is used with five degrees of freedom viz. u, v, w, θ_x and θ_y at each node. The displacement vector $\{d\}$ at any point on the mid surface is given by

$$\{d\} = \sum_{i=1}^{8} N_i(x,y)\{d_i\} \tag{5}$$

where $\{d_i\}$ and N_i are the displacement vector and the interpolating function, respectively, associated with the node i.

The virtual work equation of nonlinear dynamic equilibrium at time $t+\Delta t$ based on total Lagrangian formulation is written as [3,25]

$$\int_A \{\delta_0^{t+\Delta t} d\}^{\mathrm{T}}[\hat{\rho}]\{{}_0^{t+\Delta t}\ddot{d}\}\,\mathrm{d}A + \int_A \{\delta_0^{t+\Delta t}\varepsilon\}^{\mathrm{T}}\{{}_0^{t+\Delta t}F\}\,\mathrm{d}A = {}^{t+\Delta t}\overline{W} \tag{6}$$

where δ denotes variation, $[\hat{\rho}]$ is the inertia matrix, A is the shell area, $\{\delta_0^{t+\Delta t}\varepsilon\}$ is the variation of strain vector in the configuration at time $t+\Delta t$ referred to the configuration at time $t=0$, $\{{}_0^{t+\Delta t}F\}$ is the Cartesian component of stress vector corresponding to the configuration $t+\Delta t$ but measured in the configuration at time $t=0$ and ${}^{t+\Delta t}\overline{W}$ is the external work done.

The incremental equilibrium equation for a single element at time $t+\Delta t$ is obtained as

$$[M]\{\ddot{d}_i\} + [K]\{d_i\} = \{R\} - \{{}_0^tQ\} \tag{7}$$

where $[M]$ and $[K]$ are the mass and stiffness matrices, respectively, and $\{R\}$ and $\{Q\}$ are the external force and internal stress vectors, respectively.

Here,

$$\{{}_0^tQ\} = \int_A [B]^{\mathrm{T}}\{F\}\,\mathrm{d}A, \quad [B] = ([B_L] + [B_{NL}]) \tag{8}$$

The explicit expression of $[K]$ is

$$\begin{aligned}[K] = \int_A &([B_L]^{\mathrm{T}}[D][B_L] + [B_L]^{\mathrm{T}}[D][B_{NL}] + [B_{NL}]^{\mathrm{T}}[D][B_L] + [B_{NL}]^{\mathrm{T}}[D][B_{NL}] \\ &+ [G]^{\mathrm{T}}[S_L][G] + [G]^{\mathrm{T}}[S_{NL}][G])\,\mathrm{d}A\end{aligned} \tag{9}$$

The matrices $[S_L]$, $[S_{NL}]$, $[B_L]$, $[B_{NL}]$ and $[M]$ are given by Nanda and Bandyopadhyay [26].

2.2. Solution procedure

The solution of nonlinear equilibrium equation, Eq. (7) is implemented through an incremental iterative procedure. Eq. (7) can be written as

$$[K]\{\Delta d\}^{(i)} = \{{}^{t+\Delta t}R\} - \{{}_0^{t+\Delta t}Q\}^{(i-1)} - [M]\{{}^{t+\Delta t}\ddot{d}\}^{(i)} \quad (i = 1,2,3,\ldots) \tag{10}$$

where $\{{}^{t+\Delta t}d\}^{(i)} = \{{}^{t+\Delta t}d\}^{(i-1)} + \{\Delta d\}^{(i)}$ and $\{{}^{t+\Delta t}\ddot{d}\}^{(0)} = \{{}^t\ddot{d}\}$, $\{{}^{t+\Delta t}d\}^{(0)} = \{{}^td\}$.

The index i indicates the number of iterations. The iterative procedure is continued till $\|\{\Delta d\}^{(i)}\|/\|\{{}^{t+\Delta t}d\}^{(i-1)} + \{\Delta d\}^{(i)}\| < \gamma$ is satisfied, where γ is the tolerance for the convergence. Newmark's time integration scheme [3], known as the constant average-acceleration method, is employed to determine the nonlinear dynamic response from Eq. (10). For each time step, the modified Newton–Raphson iterations are applied to achieve equilibrium and the stiffness matrix is updated accordingly.

3. Results and discussion

3.1. Comparison problems

The program of the finite element formulation developed for the nonlinear transient analysis of laminated composite shells is validated by comparing the authors' results with those available in the existing literature for the following problems.

(1) Nonlinear transient response of simply supported composite spherical shell [13].

A simply supported cross ply (0/90) spherical shell subjected to uniformly distributed load is considered with the following geometric and material properties.

$$R_x = 10\,\text{m},\ \ a = b = 0.5\,\text{m},\ \ h = 0.01\,\text{m},$$

$$E_{11} = 25E_{22},\quad G_{12} = G_{13} = 0.5E_{22},\quad G_{23} = 0.2E_{22},\quad E_{22} = 10^6\,\text{N/cm}^2,\quad \nu_{12} = 0.25,\quad \rho = 1\,\text{Ns}^2/\text{cm}^4$$

Good agreement is observed between the authors' results of nonlinear transient response and those of Reddy and Chandrashekhara [13] from Fig. 2. It is seen that the linear response considerably differs from the nonlinear response both in the magnitude and frequency of motion. With the increase of the intensity of load, the displacement magnitude of nonlinear transient response increases and the frequency of vibration decreases.

(2) Nonlinear transient response of 0/90/90/0 simply supported plate [27].

Fig. 3 compares the authors' results of nonlinear transient response of a four-layer symmetric cross ply 0/90/90/0 simply supported plate ($E_{11} = 25E_{22},\ G_{12} = G_{13} = G_{23} = 0.5E_{22},\ E_{22} = 2.1 \times 10^6\,\text{N/cm}^2,\ \nu_{12} = 0.25$) subjected to uniform step loading with those of Chen et al. [27] employing the finite strip method. The comparison shows good agreement between the two sets of results.

Thus, the accuracy of the present formulation is established.

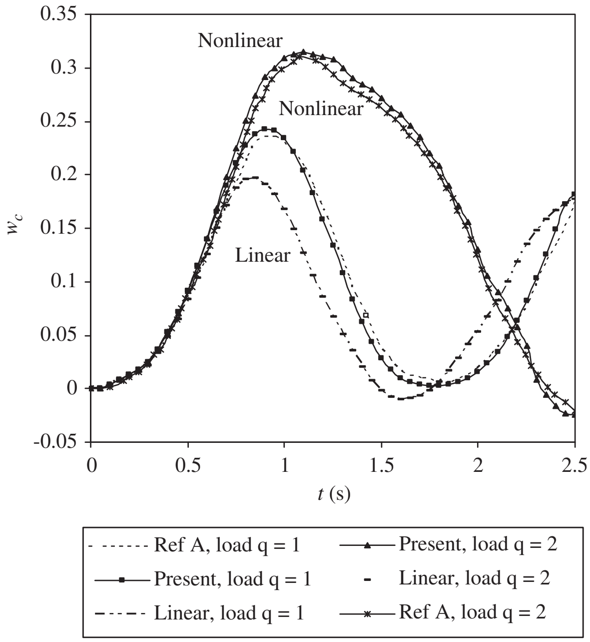

Fig. 2. Nonlinear transient response of simply supported cross ply (0/90) spherical shell subjected to uniformly distributed load (Ref A: [13], $w_c = w/q$).

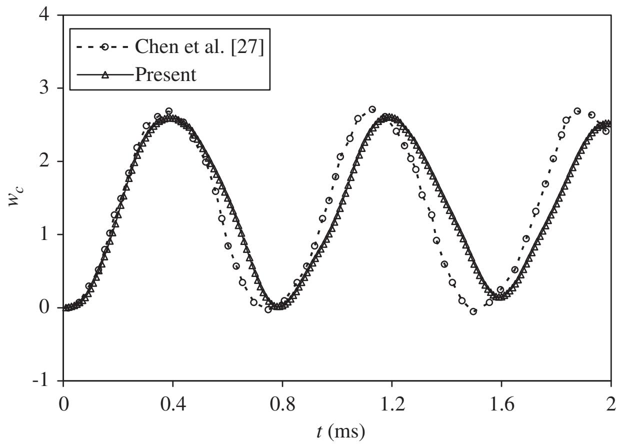

Fig. 3. Nonlinear transient response of four-layer symmetric cross ply simply supported plate subjected to uniform step loading ($a = b = 0.25$ m, $h = 0.05$ m, $w_c = w/h$).

3.2. Additional examples

Additional examples include cross ply cylindrical, spherical and hyperbolic paraboloid shells with/without cutouts and initial geometric imperfection having simply supported boundary conditions for studying the transient behavior after performing the convergence of time step. The following geometric parameters and material properties are used for the square shells ($a/b = 1$) of three forms with 0/90/0/90 lamination scheme unless mentioned otherwise:

$$R_x/b = 10, \quad b/h = 20$$

$$E_{11} = 25E_{22}, \quad G_{12} = G_{13} = 0.5E_{22}, \quad G_{23} = 0.2E_{22}, \quad v_{12} = 0.25, \quad E_{22} = 6.895\,\text{GPa}, \quad \rho = 1600\,\text{kg/m}^3$$

The critical (considering maximum positive or negative) vertical displacement (w) is expressed in non-dimensional form (w_n) as

$$w_n = wE_{22}h^3/(q_0 b^4) \times 10^4$$

where q_0 is the magnitude of uniformly distributed load.

The convergence of time step is performed taking cutout sizes (a_c/a) as 0 and 0.2 of the cylindrical shell subjected to uniformly distributed load with simply supported boundary condition. It is needless to mention that $a_c/a = 0$ refers to shell without cutout. The non-dimensional vertical displacements converge with time step of 1 ms as evident from Figs. 4 and 5. Based on the above convergence study, the time step of $\Delta t = 1$ ms is chosen for the detailed parametric study.

In the following, additional examples of cylindrical, spherical and hyperbolic paraboloid shells with a_c/a as 0 and 0.2 subjected to uniformly distributed load are taken up to study the effect of curvature and geometric imperfection.

3.2.1. Effect of radius of curvature to width ratio

In this study, radius of curvature to width ratio (R_x/b) is varied from 3 to 20. Figs. 6–8 and 9–11 present the plots of displacement versus time of laminated composite cylindrical, spherical and hyperbolic paraboloid shells with simply supported boundary condition for $a_c/a = 0$ and 0.2, respectively. A detailed study of the results of transient response from Figs. 6–11 reveals the following:

(i) Vertical displacements of spherical shell are much lower in comparison to the respective values of cylindrical and hyperbolic paraboloid shells indicating that spherical shells with and without cutouts perform better than cylindrical and hyperbolic paraboloid shells in respect of vertical displacement.

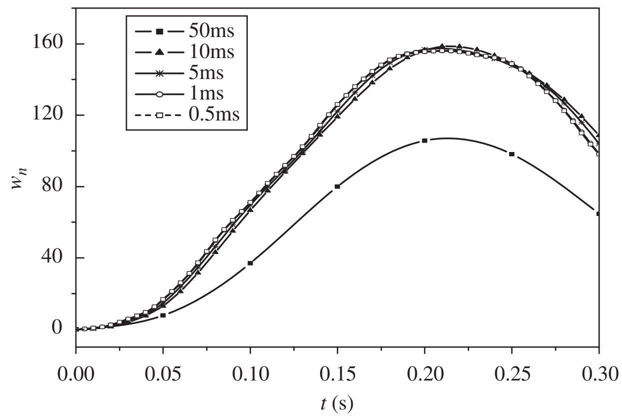

Fig. 4. Convergence of vertical displacements with respect to time step of simply supported cylindrical shell without cutout ($a_c/a = 0$).

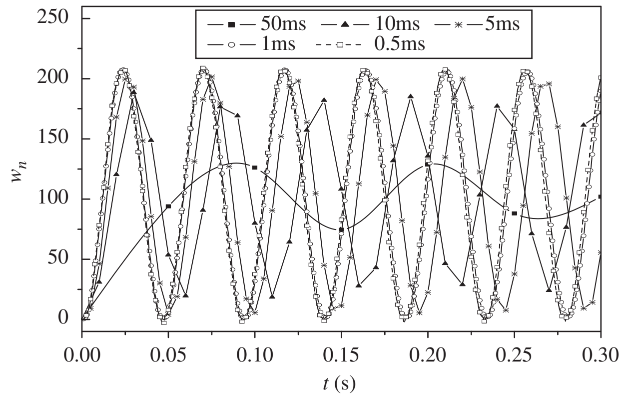

Fig. 5. Convergence of vertical displacements with respect to time step of simply supported cylindrical shell with cutout ($a_c/a = 0.2$).

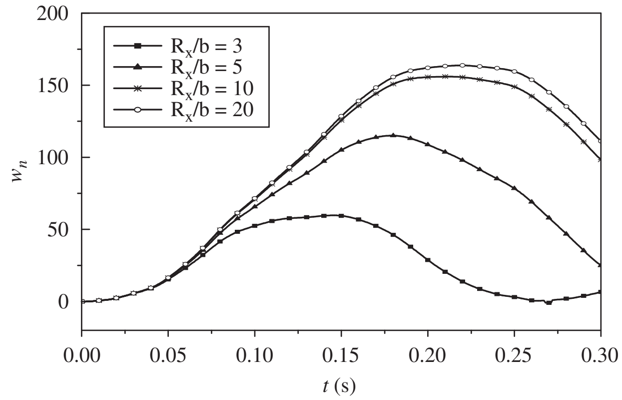

Fig. 6. Nonlinear transient response of cylindrical shell with different radius of curvature to width ratios R_x/b ($a_c/a = 0$).

(ii) The response curves for shells with cutout (Figs. 9–11) show larger number of maxima and minima in comparison to those of the shells without cutout (Figs. 6–8). Frequency of such peaks increases significantly with the introduction of cutout in these shells. Moreover, the magnitudes of displacement responses are more and cause more severity due to the introduction of cutout.

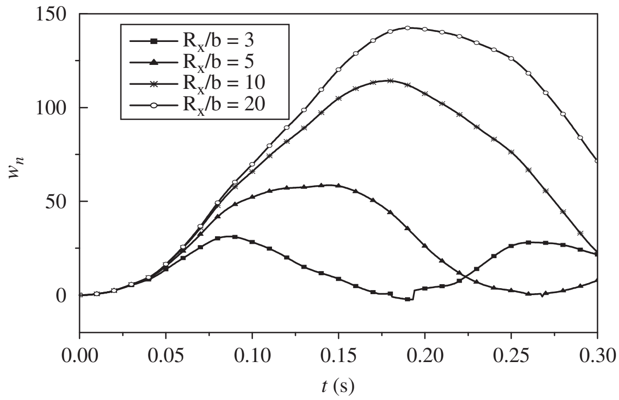

Fig. 7. Nonlinear transient response of spherical shell with different radius of curvature to width ratios R_x/b $(a_c/a = 0)$.

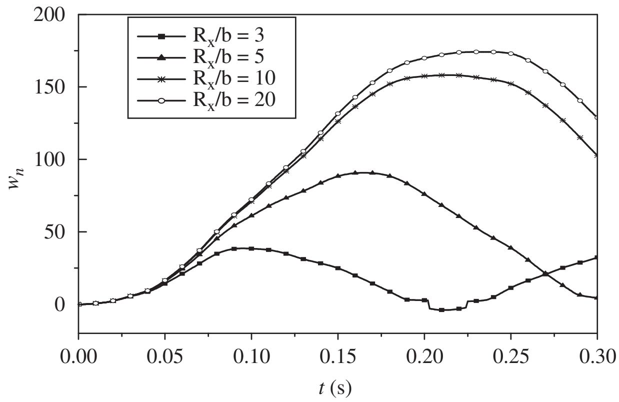

Fig. 8. Nonlinear transient response of hyperbolic paraboloid shell with different radius of curvature to width ratios R_x/b $(a_c/a = 0)$.

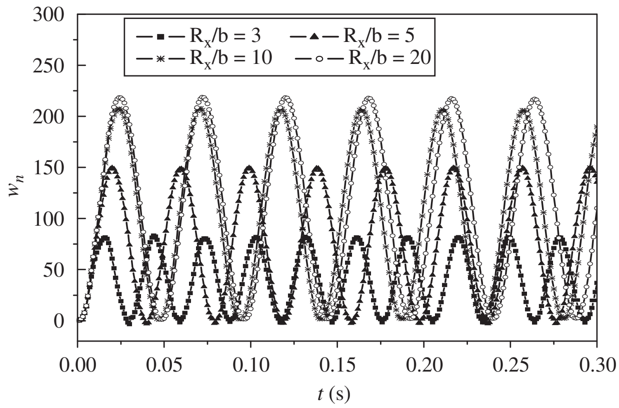

Fig. 9. Nonlinear transient response of cylindrical shell with different radius of curvature to width ratios R_x/b $(a_c/a = 0.2)$.

From the expression of the vertical displacement (Eq. (10)), it is evident that the magnitude of w_n depends on transient load, mass and stiffness of the structure. The reduction of the transient load decreases the vertical displacement, while the reductions of mass and stiffness of the structure increase the vertical displacement either individually or combinedly. The interactive effects of these three factors viz. transient load, mass and stiffness finally determine the overall increase/decrease of vertical displacement. Thus, for

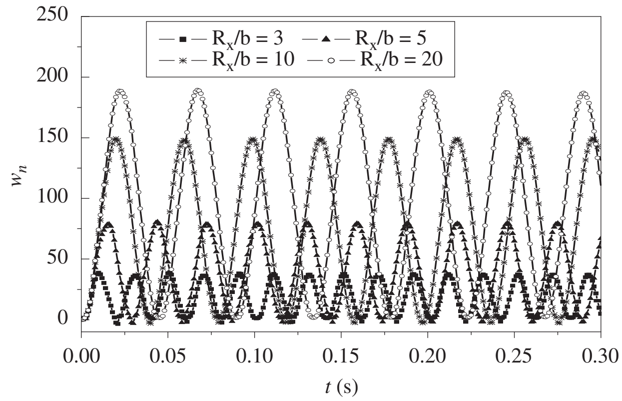

Fig. 10. Nonlinear transient response of spherical shell with different radius of curvature to width ratios R_x/b ($a_c/a = 0.2$).

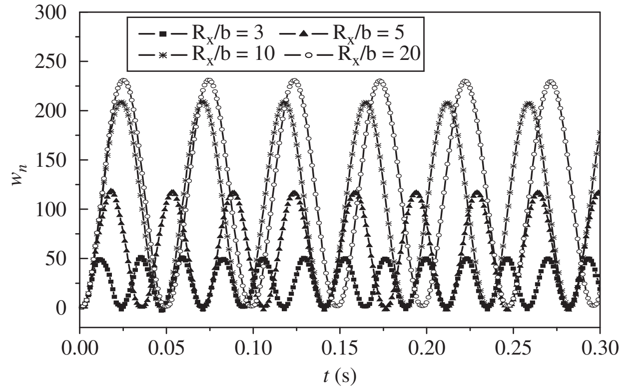

Fig. 11. Nonlinear transient response of hyperbolic paraboloid shell with different radius of curvature to width ratios R_x/b ($a_c/a = 0.2$).

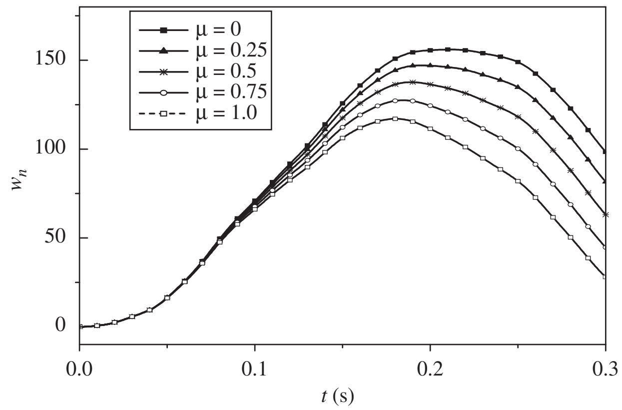

Fig. 12. Nonlinear transient response of cylindrical shell with different amplitudes of initial geometric imperfection μ ($a_c/a = 0$).

shells with cutout, the increase of the vertical displacement is due to the dominant effect of the reduction of mass and stiffness.

(iii) The vertical displacements of cylindrical, spherical and hyperbolic paraboloid shells with and without cutouts decrease with the decrease of radius to width ratio (R_x/b) indicating that the stiffness is increasing with the decrease of R_x/b. The response frequencies of them increase with the decrease of radius to width ratio. However, these effects are comparatively less when $R_x/b > 10$.

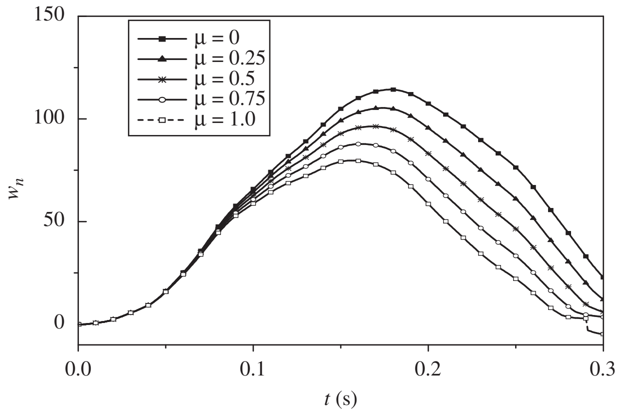

Fig. 13. Nonlinear transient response of spherical shell with different amplitudes of initial geometric imperfection μ ($a_c/a = 0$).

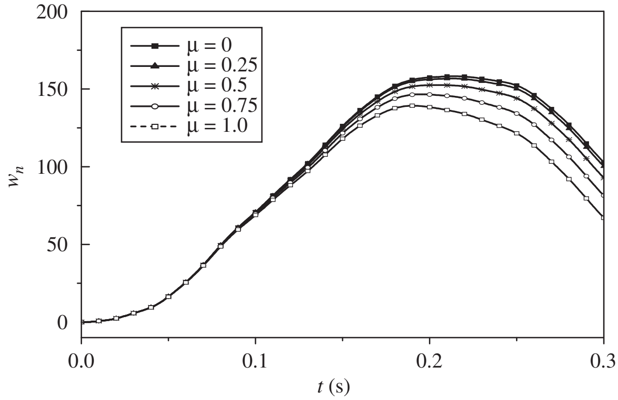

Fig. 14. Nonlinear transient response of hyperbolic paraboloid shell with different amplitudes of initial geometric imperfection μ ($a_c/a = 0$).

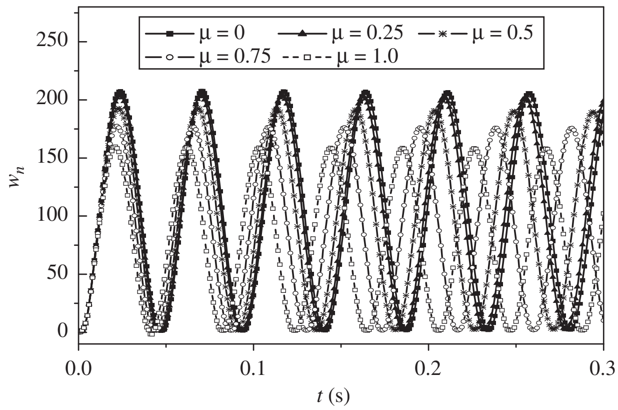

Fig. 15. Nonlinear transient response of cylindrical shell with different amplitudes of initial geometric imperfection μ ($a_c/a = 0.2$).

3.2.2. Effect of initial geometric imperfection

Here, the amplitude of initial geometric imperfection (μ) is varied from 0 to 1 to study its effect. Figs. 12–14 and 15–17 show the plots of displacement versus time of simply supported laminated composite cylindrical, spherical and hyperbolic paraboloid shells for $a_c/a = 0$ and 0.2, respectively. Detailed study of the results of

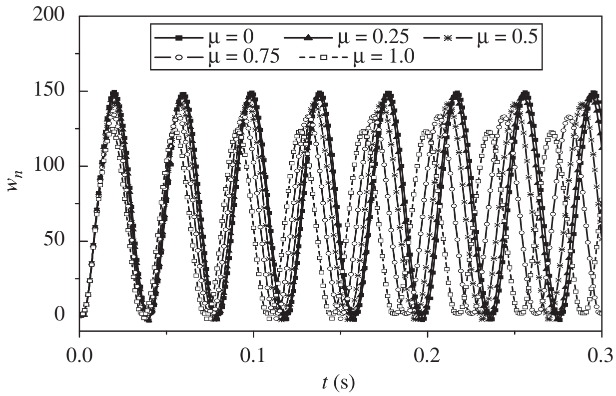

Fig. 16. Nonlinear transient response of spherical shell with different amplitudes of initial geometric imperfection μ ($a_c/a = 0.2$).

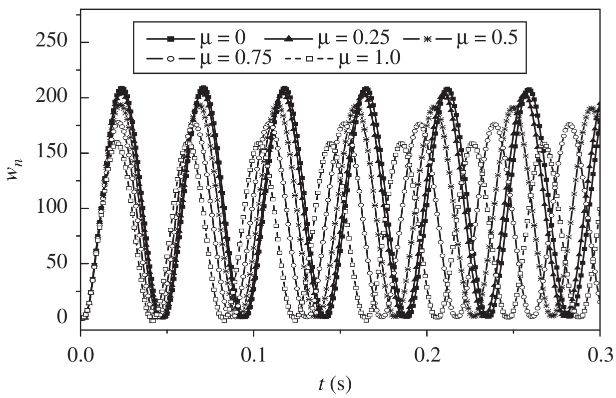

Fig. 17. Nonlinear transient response of hyperbolic paraboloid shell with different amplitudes of initial geometric imperfection μ ($a_c/a = 0.2$).

Figs. 12–17 indicates the same trend of behavior as made earlier due to the effect of radius of curvature to width ratio, shown in Figs. 6–11. It is observed from Figs. 12–17 that the vertical displacements of cylindrical, spherical and hyperbolic paraboloid shells with and without cutouts decrease and the response frequencies of them increase with the increase of amplitude of initial geometric imperfection (μ). Thus, it appears that the stiffness is increasing with the increase of μ.

4. Conclusions

The following are the summarized conclusions of the present study of transient response of cylindrical, spherical and hyperbolic paraboloid shells:

(1) The formulation developed here is recommended for the nonlinear transient analysis of laminated composite plates and shells as evident from the good agreement of the authors' results with those available in the published literature (Figs. 2–3).
(2) The authors' formulation, when applied to composite cylindrical, spherical and hyperbolic paraboloid shells with/without cutouts and geometric imperfection, leads to following conclusions:
  (a) Superiority of spherical shell with and without cutouts is established in respect of vertical displacements to cylindrical and hyperbolic paraboloid shells.
  (b) All the three forms of simply supported shells with cutout are more severe by having higher vertical displacements and response frequencies than those of shells without cutout.
  (c) For simply supported cylindrical, spherical and hyperbolic paraboloid shells with and without cutouts,

the vertical displacements decrease and the response frequencies increase with the decrease of radius to width ratio (R_x/b) possibly due to increase of stiffness. The above conclusion of decreasing vertical displacements and increasing response frequencies is also applicable for increase of amplitude of initial geometric imperfection. Here also, the increase of stiffness of the structure is possibly reducing the vertical displacement and increasing the response frequencies.

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